# Framework for piloting for methods in mathematics

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# 1 Introduction

GCSE subject criteria set out the knowledge, understanding, skills and assessment objectives common to all GCSE specifications in methods in mathematics**.** They provide the framework within which an awarding body creates the detail of the specification.

Specifications must also meet the regulators' general requirements, including the common and GCSE criteria as defined in *The statutory regulation of external qualifications* (QCA/04/1293).

Subject criteria are intended to:

- help ensure consistent and comparable standards in the same subject across the awarding bodies
- ensure that the rigour of GCSE is maintained
- ensure that specifications build on the knowledge, understanding and skills established by the national curricula for England, Northern Ireland and Wales, and facilitate progression to higher level qualifications in mathematics
- help higher education institutions, employers and other stakeholders, such as learners and parents/guardians, know what has been studied and assessed.

Any GCSE specification that contains significant elements of methods in mathematics must be consistent with the relevant parts of these subject criteria.

# 2 Aims and learning outcomes

GCSE specifications in methods in mathematics should encourage learners to be inspired, moved and changed by following a broad, coherent, satisfying and worthwhile course of study. They should help learners to develop confidence in, and a positive attitude towards, mathematics and to recognise the importance of mathematics in their own lives and to society. They should enable learners to make informed judgements about the use of technology. Specifications should enable learners to appreciate the coherence, creativity, elegance and power of mathematics. They should prepare learners to make informed decisions about further learning opportunities and career choices.

Specifications in methods in mathematics must enable learners to:

- develop knowledge, skills and understanding of mathematical methods, techniques and concepts

- make connections between different areas of mathematics
- select and apply mathematical methods in mathematical contexts
- reason mathematically, construct arguments and simple proofs, and make logical deductions and inferences
- develop and refine strategies for solving a range of mathematical problems
- communicate mathematical information in a variety of forms.

# 3 Subject content

The content of GCSE specifications in methods in mathematics must reflect the learning outcomes.

GCSE specifications in methods in mathematics must be consistent with the national curriculum key stage 4 programmes of study requirements for mathematics in the orders for England and Wales, and the statutory requirements for key stage 4 in Northern Ireland.

In combination with GCSE in applications of mathematics, GCSE specifications in methods in mathematics must meet the requirements of the national curriculum key stage 4 programmes of study requirements for mathematics in the orders for England and Wales in full.

GCSE specifications in methods in mathematics must enable learners to develop the knowledge, skills and understanding specified below. **Higher tier content is in bold type.**

## 3.1 Number

- Understand and use number operations and the relationships between them, including inverse operations and hierarchy of operations.
- Arithmetic of real numbers, including **exact calculation with surds and pi.**
- Numbers and their representations including powers, roots, indices (integers, **fractional and negative)**, and **standard index form.**
- Approximate to specified or appropriate degrees of accuracy including a given power of ten, number of decimal places and significant figures.
- Use the concepts and vocabulary of factor (divisor), multiple, common factor, common multiple, highest common factor, least common multiple, prime number and prime factor decomposition.

- Understand that factors of a number can be derived from its prime factorisation.
- Understand that 'percentage' means 'number of parts per 100' and use this to compare proportions.
- Use multipliers for percentage change**; work with repeated percentage change; solve reverse percentage problems.**
- Interpret fractions, decimals and percentages as operators.
- Understand and use the relationship between fractions and decimal representations including recurring and terminating decimals.
- Understand and use the relationship between ratio and fractions.
- Find proportional change and **repeated proportional change,** using fractions, decimals and percentages.
- Understand and use direct and **inverse** proportion.
- Divide a quantity in a given ratio.
- Use calculators effectively and efficiently, including **trigonometrical** functions.
- Understand and use Venn diagrams to solve problems.

## 3.2 Algebra

- Distinguish the different roles played by letter symbols in algebra, using the correct notation.
- Distinguish in meaning between the words equation, inequality, formula, **identity** and expression.
- Manipulate algebraic expressions by collecting like terms, by multiplying a single term over a bracket, taking out common factors, **multiplying two linear expressions, factorising quadratic expressions including the difference of two squares, and simplifying rational expressions.**
- Set up, and solve simple equations and inequalities.
- **Set up and use equations that describe direct and inverse proportion.**
- **Set up, and solve simultaneous equations in two unknowns where one of the equations might include squared terms in one or both unknowns.**
- Solve quadratic equations approximately using a graph, **exactly by factorising, completing the square and using the formula.**

- Derive a formula, substitute numbers into a formula and change the subject of a formula.
- Generate terms of a sequence using term-to-term and position-to-term definitions.
- Form linear **and quadratic** expressions to describe the *n*th term of a sequence.
- Use algebra to support and construct arguments **and proofs.**
- Use the conventions for coordinates in the plane and plot points in all four quadrants.
- Recognise and plot equations that correspond to straight-line graphs in the co-ordinate plane.
- Use geometric information to complete diagrams on a co-ordinate grid.
- **Use *y* = *mx* + *c* and understand the relationship between gradients of parallel and perpendicular lines,**
- **Draw, sketch, recognise graphs of linear, quadratic simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, the function $y = k^x$ for integer values of *x* and simple positive values of *k*, the trigonometric functions *y* = sin *x*, *y* = cos *x and y = tan x***
- **Understand and use the Cartesian equation of a circle centred at the origin and link to the trigonometric functions.**
- **Construct the graphs of simple loci.**
- **Sketch simple transformations of a given function.**
- Recognise and use equivalence in numerical, algebraic and graphical representations.

## 3.3 Geometry

- Recall and use properties of angles at a point, angles at a point on a straight line (including right angles), perpendicular lines, and vertically opposite angles.
- Understand and use the angle properties of parallel and intersecting lines, triangles and quadrilaterals.
- Recall the properties and definitions of special types of quadrilateral, including square, rectangle, parallelogram, trapezium, kite and rhombus.

- Recognise reflection and rotation symmetry of 2D shapes.
- Calculate and use the sums of the interior and exterior angles of polygons.
- Solve problems in the context of tiling patterns and tessellation.
- **Understand, prove and use circle theorems, intersecting chords.**
- **Understand and use the midpoint and the intercept theorems.**
- **Understand and construct geometrical proofs using formal arguments, including proving the congruence, or non congruence of two triangles in all possible cases.**
- Describe and transform 2D shapes using single or combined rotations, reflections, translations, or enlargements by a positive scale factor **then use positive fractional and negative scale factors** and distinguish properties that are preserved under particular transformations.
- Use 2D vectors to describe translations.
- **Use vectors to solve simple geometric problems and construct geometric arguments.**
- Understand congruence and similarity, including the relationship between lengths, **areas and volumes** in similar figures.
- Use Pythagoras' theorem in 2D **and 3D**.
- **Use the trigonometrical ratios to solve 2D and 3D problems.**
- **Use the sine and cosine rules to solve problems in 2D and 3D.**
- Distinguish between centre, radius, chord, diameter, circumference, tangent, arc, sector and segment.
- Find circumferences of circles and areas enclosed by circles.
- Calculate perimeters and areas of shapes made from triangles and rectangles **and other shapes**.
- **Calculate the area of a triangle using** $\frac{1}{2}ab\sin C$
- Calculate volumes of right prisms and of shapes made from cubes and cuboids.
- **Solve mensuration problems involving more complex shapes and solids.**

### 3.4 Probability

- Understand and use the vocabulary of probability and the probability scale.
- **Understand and use theoretical models for probabilities including the model of equally likely outcomes.**
- **Understand and use estimates of probability from relative frequency.**
- Use of sample spaces for situations where outcomes are single events and for situations where outcomes are two successive events.
- Identify different mutually exclusive and exhaustive outcomes and know that the sum of the probabilities of all these outcomes is 1.
- Understand and use set notation to describe events and compound events.
- Use Venn diagrams to represent the number of possibilities and hence find probabilities.
- **Use tree diagrams to represent outcomes of compound events, recognising when events are independent or dependent.**
- Know when to add or multiply probabilities: if $A$ and $B$ are mutually exclusive, then the probability of $A$ or $B$ occurring is $P(A) + P(B)$; if $A$ and $B$ are independent events, the probability of $A$ and $B$ occurring is $P(A) \times P(B)$.
- Compare experimental data and theoretical probabilities, and make informal inferences about the validity of the model giving rise to the theoretical probabilities.
- Understand that when a statistical experiment or survey is repeated there will usually be different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.

# 4 Assessment objectives

All specifications in methods in mathematics must require learners to demonstrate their ability to:

| Assessment objectives | | Weighting (%) |
|---|---|---|
| **AO1** | Recall and use their knowledge of the prescribed content | **50–60** |

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| **AO2** | Select and apply mathematical methods | **15–25** |
| **AO3** | Interpret and analyse problems and use mathematical reasoning to solve them | **20–30** |

# 5 Scheme of assessment

GCSE specifications in methods in mathematics must allocate a weighting of 100 per cent to external assessment.

Question papers in methods in mathematics must be targeted at either foundation or higher tier.

Each scheme of assessment must allocate a minimum weighting of 25 per cent, and a maximum weighting of 50 per cent, to assessment without a calculator.

# 6 Grade descriptions

Grade descriptions are provided to give a general indication of the standards of achievement likely to have been shown by candidates awarded particular grades. The descriptions must be interpreted in relation to the content in the specification; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of candidates' performance in the assessment may be balanced by better performances in others.

| Grade | Description |
|---|---|
| **A** | Candidates use a wide range of mathematical techniques, terminology, diagrams and symbols consistently, appropriately and accurately. Candidates are able to use different representations effectively and they recognise equivalent representations for example numerical, graphical and algebraic representations. Their numerical skills are sound, they use a calculator effectively and they demonstrate algebraic fluency. They use trigonometry and geometrical properties to solve problems.<br><br>Candidates identify and use mathematics accurately in a range of |

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| | contexts. They evaluate the appropriateness, effectiveness and efficiency of different approaches. Candidates choose methods of mathematical communication appropriate to the context. They are able to state the limitations of an approach or the accuracy of results. They use this information to inform conclusions within a mathematical or statistical problem.<br><br>Candidates make and test hypotheses and conjectures. They adopt appropriate strategies to tackle problems (including those that are novel or unfamiliar), adjusting their approach when necessary. They tackle problems that bring together different aspects of mathematics and may involve multiple variables. They can identify some variables and investigate them systematically; the outcomes of which are used in solving the problem.<br><br>Candidates communicate their chosen strategy. They can construct a rigorous argument, making inferences and drawing conclusions. They produce simple proofs and can identify errors in reasoning. |
| **C** | Candidates use a range of mathematical techniques, terminology, diagrams and symbols consistently, appropriately and accurately. Candidates are able to use different representations effectively and they recognise some equivalent representations eg numerical, graphical and algebraic representations of linear functions; percentages, fractions and decimals. Their numerical skills are sound and they use a calculator accurately. They apply ideas of proportionality to numerical problems and use geometric properties of angles, lines and shapes.<br><br>Candidates identify relevant information, select appropriate representations and apply appropriate methods and knowledge. They are able to move from one representation to another, in order to make sense of a situation. Candidates use different methods of mathematical communication.<br><br>Candidates tackle problems that bring aspects of mathematics together. They identify evidence that supports or refutes conjectures and hypotheses. They understand the limitations of evidence and sampling, and the difference between a mathematical argument and conclusions based on experimental evidence.<br><br>They identify strategies to solve problems involving a limited number of variables. They communicate their chosen strategy, making changes as necessary. They construct a mathematical argument and identify inconsistencies in a given argument or exceptions to a |

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| | generalisation. |
| **F** | Candidates use some mathematical techniques, terminology, diagrams and symbols from the foundation tier consistently, appropriately and accurately. Candidates use some different representations effectively and can select information from them. They complete straightforward calculations competently with and without a calculator. They use simple fractions and percentages, simple formulae and some geometric properties, including symmetry.<br><br>Candidates work mathematically in everyday and meaningful contexts. They make use of diagrams and symbols to communicate mathematical ideas. Sometimes, they check the accuracy and reasonableness of their results.<br><br>Candidates test simple hypotheses and conjectures based on evidence. Candidates are able to use data to look for patterns and relationships. They state a generalisation arising from a set of results and identify counter-examples. They solve simple problems, some of which are non-routine. |

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